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# Event-triggered Coordination for Formation Tracking Control in Constrained Space with Limited Communication

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**Abstract**—In this paper, the formation tracking control is studied for a multi-agent system (MAS) with communication limitations. The objective is to control a group of agents to track a desired trajectory while maintaining a given formation in non-omniscient constrained space. The role switching triggered by the detection of unexpected spatial constraints facilitates efficiency of event-triggered control in communication bandwidth, energy consumption and processor usage. A coordination mechanism is proposed based on a novel role ‘coordinator’ to indirectly spread environmental information among the whole communication network and form a feedback link from followers to the leader to guarantee the formation keeping. A formation scaling factor is introduced to scale up or scale down the given formation size in the case that the region is impassable for MAS with the original formation size. Controllers for the leader and followers are designed and the adaptation law is developed for the formation scaling factor. The conditions for asymptotic stability of MAS are discussed based on the Lyapunov theory. Simulation results are presented to illustrate the performance of proposed approaches.

**Keywords:** multi-agent coordination, constrained space, formation tracking control, event-triggered control, formation scaling, communication limitations

## I. INTRODUCTION

In recent years, there has been tremendous interest in cooperative control of multi-agent systems (MAS) [1], [2], [3], [4], [5]. Especially, formation control has attracted considerable attention from researchers. Formation control is defined as the coordination of multiple agents that enter into and maintain a specific formation. Potential application areas of formation control include cooperative tasks such as exploration, surveillance, search and rescue, transportation of large objects and control of arrays of satellites [6], [7], [8], [9].

Leader-follower formation control of MAS has been widely studied recently [10], [11], [13], where the leader tracks a desired path and the follower maintains a predefined geometric configuration with the leader. The fact that only a single group leader is involved in the team relaxes the requirement of

communication bandwidth and simplifies the implementation of the approach. However, the overreliance on the leader and blind follow may increase the risk of task failure caused by false decision made by the leader or the break of communication link between the leader and followers. Another issue with the typical leader-follower approach is lack of internal information feedback throughout the group, which weakens the coordination among agents.

The significance of adjustable leader/follower roles for shared control has been emphasized in a recent review in the field of human-robot interaction [14], and there are several works in this direction [15], [16], [17]. Such a human-robot system is formulated as a two-agent system with one leader and one follower. Similarly, in displacement-based control of MAS [18], [19], [20], given a task of trajectory tracking, only the leader has the knowledge of the desired trajectory while the followers are aware of the displacements with respect to the leader to achieve the desired formation. In this structure, since only the followers track the leader and there is no feedback from the followers to the leader, if a follower fails to follow properly, no mechanism can guarantee the formation keeping. Therefore, this issue challenges the success of formation control especially in unknown spatially constrained environments. In [21], Abdollahi and Rezaee proposed a technique based on the behavioral structure and designed an approach to avoid the obstacle by applying the rotational potential field. In this study, agents were designed to avoid collisions with obstacles individually, which means that the coordination was broken temporarily during the process of collision avoidance, i.e., if any agent failed to follow, then the original desired formation had to be given up. In the situation where agents do not have accurate global environmental information, they have to avoid collisions with unexpected spatial constraints. In this regard, the introduction of role switching for MAS is significant which provides a coordination mechanism for collision avoidance.

The topic of cooperative control in restricted space has attracted increasing attention from researchers in different areas and many achievements have been reported [22], [23], [24], [25]. In [26], a scaling matrix was introduced to scale up or scale down the specific geometric formation shape depending on the given task in a bounded region. In this approach, two objective functions have to be constructed respectively for outer sub-region and inner sub-region but no guidelines were provided for the construction. The simulation examples assumed the formation center to be approximately the center

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of the constrained region and in the meantime approximated the bounded region as an ellipse. In [27], the formation coordination was studied with fixed shape and variable size. The implementation of formation scaling strategy requires only local relative position information based on environmental conditions, mission commands, etc. In [28], Lu et al. studied control of a group of mobile agents to have a desired formation while flocking in a constrained environment. There was no tracking requirement in this study, which demands a higher level of coordination mechanism for MAS.

To address the issue of unexpected multiple spatial constraints in a non-omniscient bounded space, the cooperative formation tracking control of MAS with communication constraints including limited communication range and limited communication bandwidth is studied. Followers are expected to achieve and maintain the desired formation with the knowledge of neighbors' states, which locate within the communication range of the follower. The detection of unexpected spatial constraints will trigger the coordination mode for collision avoidance. In this situation, the followers in the influence range of the obstacles and the leader will switch into the role of coordinator. The proposal of role switching is motivated by benefits of event-triggered control on efficiency in network resources as well as energy. The size of formation shape can be scaled up or down in the case that the path is impassable for the MAS with the original formation size.

The main contribution of this paper is the proposal of a coordination solution under the frame of event-triggered role switching control for the formation tracking of MAS in constrained space with communication limitations. The detection of unexpected spatial constraints will trigger the role switching, which also switches the MAS from the leader tracking mode to the coordination mode for collision avoidance for the sake of efficiency in communication bandwidth, energy consumption and processor usage. In the coordination mode, a novel concept of coordinator is introduced to provide a feedback link from followers to the leader for better mutual collaboration guaranteeing the success of formation keeping. Collisions can be cooperatively avoided by only broadcasting agents' state, while the state of spatial constraints does not need to be broadcasted. Furthermore, a formation scaling factor is introduced concerning the circumstance that the region is impassable for MAS with the original formation size and an adaptation law for formation scaling factor is designed to be implemented in real time without any apriori knowledge about the environment. The conditions for asymptotic stability of MAS are extracted based on the Lyapunov theory to indicate whether the formation tracking task should be given up or switching the formation.

The remainder of the paper is organized as follows. In Section II, the problem is formulated and some preliminaries are presented. In Section III, the concept of coordinator is proposed and controllers are designed for different agents to generate and keep formation in a constrained space. In Section IV, the performance of the formation tracking controller for MAS is analyzed based on the Lyapunov theory and the adaptation law for the formation scaling factor is developed. In Section V, simulation results are presented to demonstrate the

performance of the proposed schemes. Finally, we conclude this paper in Section VI.

## II. SYSTEM DYNAMICS AND FORMATION MANEUVERS

### A. Problem Statement

Consider a network of multi-agent systems  $\mathcal{V} = \{v_1, \dots, v_N\}$ , where  $v_i, i = 1, \dots, N$  represents  $i^{th}$  agent in the network and  $N$  is the number of agents. The following assumptions are made in this paper.

**Assumption 1.** The influence of the size and the shape of an agent to the formation tracking control are ignored, which means that an agent is assumed to be a point mass.

**Assumption 2.** An agent is able to estimate its position in the world coordinate system.

**Assumption 3.** Without loss of generality, it is assumed that the first agent  $v_1$  is the leader for trajectory tracking, who knows the desired reference while other agents are unaware of the desired reference but know which agent is the leader.

Denote  $x_i(t) \in \mathbb{R}^m$  as the position of agent  $v_i$  at time  $t$ , where  $m = 2$  or  $m = 3$ . The formation pattern at time  $t$  is defined to be a set of desired displacement  $\mathcal{P} = \{\delta_2, \dots, \delta_N\}$  for followers  $v_i, i = 2, \dots, N$ , where  $\delta_i$  is the desired displacement of the agent  $v_i$  relative to the leader  $v_1$ . Then we have  $x_i^d(t) = x_1(t) + \delta_i$ , where  $x_i^d$  is the desired formation position of agent  $v_i$  at time  $t$ , and write the desired displacement between  $x_i$  and  $x_j$  as  $\delta_{ij}$ , such that  $\delta_{ij} \triangleq x_i^d(t) - x_j^d(t) = \delta_i - \delta_j$ . We assume  $N$  agents with the similar dynamics described by the following linear equation

$$\dot{x}_i(t) = u_i(t) \quad (1)$$

where  $u_i(t)$  represents the control vector of agent  $v_i$ . Denote the reference trajectory known only by the leader  $v_1$  as  $y^d(t)$  at time  $t$ , then the trajectory tracking error for the leader  $v_1$  can be written as  $\varepsilon_1 \triangleq x_1 - y^d$ , while the formation error for follower  $v_i, i = 2, \dots, N$  is  $\varepsilon_i \triangleq x_i - x_i^d = x_i - x_1 - \delta_i$ .

**Assumption 4.** There exists a constant  $\zeta_1$  such that  $0 \leq \|y^d\| \leq \zeta_1 < \infty$ .

**Control objective:** Design a controller for a group of agents initialized on random positions to track the desired trajectory in formation in a non-omniscient constrained space. In other words, the center of agents and the center of the desired formation should eventually coincide. In the meantime, consider the unknown forbidden space as  $\Pi$ , which means that all agents' positions satisfy the spatial constraint condition as below

$$x_i(t) \notin \Pi \quad i = 1, \dots, N, 0 \leq t < \infty \quad (2)$$

during the whole task. The control goal can be described as:  $\forall t \geq 0, \sum_{i=1}^N \|\varepsilon_i(t)\| \leq \epsilon$ , where  $\epsilon$  is a positive constant.

**Case 1:** when  $\Pi = \emptyset$ ,  $\epsilon$  can be made sufficiently small when  $t \rightarrow \infty$ , such that  $\lim_{t \rightarrow \infty} \sum_{i=1}^N \|\varepsilon_i(t)\| = 0$ .

**Case 2:** when  $\Pi \neq \emptyset$ ,

$$\epsilon \propto \frac{1}{\rho(x_i, \Pi)}, \quad \text{and} \quad \epsilon \leq \epsilon_{max} \quad (3)$$

where  $\rho(x_i, \Pi) = \inf_{s \in \Pi} \rho(x_i, s)$ .  $\rho(\cdot, \cdot)$  is defined as Euclidean distance in this paper, and  $\epsilon_{max}$  indicates tolerable upper limit for the formation error.

### B. Graph Theory

A team of agents interact with each other via communication or sensing networks to achieve collaborative objectives. It is convenient to model the information exchanges among agents by undirected graphs. An undirected graph  $\mathcal{G}$  is a pair  $(\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V} = \{v_1, \dots, v_N\}$  is a nonempty finite node set and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  is an edge set of ordered pairs of nodes, called edges. The adjacency matrix  $A = [a_{ij}] \in \mathbb{R}^{N \times N}$  associated with undirected graph  $\mathcal{G}$  is defined such that

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{ \& } (v_i, v_j) \in \mathcal{E} \text{ (or } (v_j, v_i) \in \mathcal{E}) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Especially, the adjacency vector of the leader is written as  $B = [b_i] \in \mathbb{R}^{N \times 1}$  such that  $b_i = a_{i1} = a_{1i}$  and

$$b_i = \begin{cases} 1 & \text{if } i \neq 1 \text{ \& } (v_i, v_1) \in \mathcal{E} \text{ (or } (v_1, v_i) \in \mathcal{E}) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

The Laplacian matrix  $L = [L_{ij}] \in \mathbb{R}^{N \times N}$  of graph  $\mathcal{G}$  is defined as  $L_{ii} = \sum_{j \neq i} a_{ij}$  and  $L_{ij} = -a_{ij}$ ,  $i \neq j$ . The Laplacian matrix can be written into a compact form as  $L = D - A$ , where  $D = \text{diag}(d_1, \dots, d_N)$  is the degree matrix with  $d_i$  as the in-degree of the  $i$ -th node.

**Assumption 5.** The subgraph  $\mathcal{G}_s$  associated with the followers is undirected and in the graph  $\mathcal{G}$  the leader has directed paths to all followers. Equivalently,  $\mathcal{G}$  contains a directed spanning tree with the leader as the root.

## III. ROLE SWITCHING FOR MULTI-AGENTS COORDINATION

The following definitions of roles in the task of formation tracking in a constrained space are made first.

**Definition 1 (Leader).** In MAS, the agent which has the knowledge of the desired trajectory or whose behavior is influenced by only the environment is defined as a *Leader*.

**Definition 2 (Coordinator).** In MAS, the agent whose behavior is influenced by both the environment and its neighbors is defined as a *Coordinator*.

**Definition 3 (Follower).** In MAS, the agent whose behavior is influenced by only its neighbors is defined as a *Follower*.

### A. Trajectory Tracking & Formation Following

On the one hand, based on Assumption 3, the first agent knows the reference trajectory  $y^d(t)$ , so the control for the first agent can be directly designed by using the certainty equivalence principal to track  $y^d(t)$ . The controller is given as

$$u_1 = -k_t \varepsilon_1 + \dot{y}^d \quad (8)$$

where  $k_t$  is a positive scalar.

On the other hand, the follower  $v_i, i = 2, \dots, N$ , has no information about the desired trajectory, and it tracks the leader

in formation with the knowledge of its neighbors' positions. The controller for  $v_i$  is proposed as

$$u_i = -k_n \frac{b_i}{d_i} (x_i - x_1 - \delta_i) - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \delta_{ij}) - k_g g \left( \frac{b_i}{d_i} (x_i - x_1 - \delta_i) + \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \delta_{ij}) \right) \quad (9)$$

where  $k_n > 0$  and  $k_g > 0 \in \mathbb{R}$  are two scalars, and  $g(\cdot)$  is a nonlinear function such that for  $\omega \in \mathbb{R}^m$ ,

$$g(\omega) = \begin{cases} \frac{\omega}{\|\omega\|} & \text{if } \|\omega\| \neq 0 \\ 0 & \text{if } \|\omega\| = 0 \end{cases} \quad (10)$$

$$(11)$$

### B. Spatial Collision Avoidance

In this subsection,  $U_i^c(x)$  is designed to avoid collisions with spatial constraints. Let  $\Pi = \Pi_1 \cup \Pi_2 \dots \cup \Pi_R$ , where  $R$  is the number of continuous constrained spaces. The artificial potential field  $U_i^c(x)$  designed to avoid spatial collisions can be written as

$$U_i^c(x_i) = \sum_{r=1}^R \int \frac{\delta(\|\mathfrak{S}_{ir}\|)}{\mathfrak{X}_i} ds \quad (12)$$

where  $\int ds$  is the simplification for  $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ds_1 ds_2$  in 2-D space and  $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ds_1 ds_2 ds_3$  in 3-D space.  $\mathfrak{S}_{ir}$  represents a specific condition to pick force points at the spatial constraint  $\Pi_r$  ( $r = 1, \dots, R$ ) and  $\mathfrak{X}_i = \|x_i - s\| - s_0$ , where  $s_0$  is a small constant regarded as the safety distance to avoid collisions with constraints edges.  $\delta$  indicates Dirac delta function [29] utilized to integrate the potential force from all points on edges of spatial constraints satisfying the specific condition. The virtual force generated accordingly is

$$F_i^c = -\nabla U_i^c = \sum_{r=1}^R \int \frac{\delta(\|\mathfrak{S}_{ir}\|)(x_i - s)}{\mathfrak{X}_i^2 \|x_i - s\|} ds \quad (13)$$

The derivation from the artificial potential field in Eq.(12) to virtual force in Eq.(13) ignores impulsive influence from spatial constraints. This issue is not the focus of this paper, we will not elaborate it in this paper. Readers may refer to [31] for detailed discussion.

**Remark 1.** Artificial potential field (APF) method provides simple and effective solutions for practical applications, and has been used extensively to produce feedback control laws. The application of APF for obstacle avoidance was first developed in [30]. The basic idea is to design a potential field which “repulses” the agent from obstacles, while the control force is derived by the negative gradient of the joint potential to extract the control scheme.

We define that  $U_i^c > \mu_0$  as the trigger condition for role switching, where  $\mu_0 > 0$  is a constant to reduce the influence of noise and false alarm. It reflects the detection of obstacles and the potential of collisions. In the case that any follower  $v_i, i = 2, \dots, N$  satisfies the condition, MAS will switch to

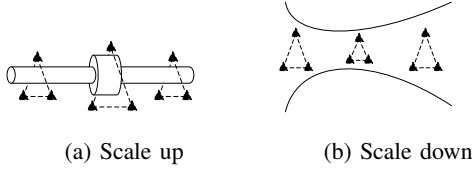


Fig. 1: Two examples of formation scaling

the coordination mode for collision avoidance from the leader tracking mode. The controllers of the leader (which has been downgraded to be a coordinator) and followers become

$$u_1 = k_c F_1^c + \dot{y}^d - k_t \varepsilon_1 - k_n \sum_{i=2}^N \frac{b_i}{d_1} (x_1 - x_i + \delta_i) \times \operatorname{sgn} \left( \sum_{i=2}^N \frac{b_i}{d_1} (x_1 - x_i + \delta_i)^T (k_c F_1^c - k_t \varepsilon_1) \right) \quad (14)$$

$$u_i = k_c F_i^c - k_n \frac{b_i}{d_i} (x_i - x_1 - \delta_i) - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \delta_{ij}) - k_g g \left( \frac{b_i}{d_i} (x_i - x_1 - \delta_i) + \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \delta_{ij}) \right) \quad (15)$$

where  $k_c > 0$  is a scalar and  $\operatorname{sgn}(\cdot)$  is the sign function such that

$$\operatorname{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

In this situation, the leader downgrades to a coordinator to receive the information of spatial constraints detected by other agents and then locally adjusts the original trajectory to achieve collision avoidance. Meanwhile, some followers  $\{v_i | F_i^c \neq 0, i = 2, \dots, N\}$  upgrade to coordinators to spread the influence of spatial constraints to communication network of MAS such that the given formation can be achieved while avoiding collisions with obstacles. Comparing Eq.(14) with Eq.(8) and concerning the system dynamics in Eq.(1), the first item is to introduce the effect of spatial constraints to the agent's velocity for collision avoidance, while the last item is to provide a feedback from neighbor agents on the completeness of the formation following. The existence of spatial constraints that influences the formation following will indirectly affect the motion of the leader to cooperatively pass the constrained region.

### C. Formation Scaling

In a realistic environment, the restricted path may not always be passable for MAS with the given formation size. To get through the path without collisions, we improve formation

control laws with size scaling as below

$$u_1 = k_c F_1^c + \dot{y}^d - k_t \varepsilon_1 - k_n \sum_{i=2}^N \frac{b_i}{d_1} (x_1 - x_i + \lambda_i \delta_i) \times \operatorname{sgn} \left( \sum_{i=2}^N \frac{b_i}{d_1} (x_1 - x_i + \lambda_i \delta_i)^T (k_c F_1^c - k_t \varepsilon_1) \right) \quad (16)$$

$$u_i = k_c F_i^c - k_n \frac{b_i}{d_i} (x_i - x_1 - \lambda_i \delta_i) - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \lambda_j \delta_j) - k_g g \left( \frac{b_i}{d_i} (x_i - x_1 - \lambda_i \delta_i) + \sum_{j=2}^N \frac{a_{ij}}{d_i} (x_i - x_j - \lambda_j \delta_j) \right) \quad (17)$$

In above controller, a time-varying scaling factor  $\lambda_i(t)$  is introduced to adjust the size of the given geometric formation shape initialized as 1 for agent  $v_i$ . Define  $\hat{\varepsilon}_i$  as the scaling formation error for agent  $v_i, i = 2, \dots, N$ , such that

$$\hat{\varepsilon}_i \triangleq x_i - x_1 - \lambda_i \delta_i \quad (18)$$

Two instances where the formation scaling is required to enable MAS to pass the restricted region are described in Fig.1. In the first case (see Fig.1a), the size of formation should be scaled up; while in the second case (see Fig.1b), the size of formation should be scaled down. Let

$$\Upsilon = \begin{bmatrix} \frac{1}{d_2} & 0 & \cdots & 0 \\ 0 & \frac{1}{d_3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_N} \end{bmatrix} \quad (19)$$

and define  $H = \Upsilon(L + B)$ . The update rule for scaling factor  $\lambda_i, i = 2, \dots, N$  is introduced as Eq.(20),

$$\dot{\lambda}_i = \begin{cases} \alpha \operatorname{sgn}(F_i^{cT} \delta_i) & \text{if } \tau_1 \leq F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j \text{ \& } \underline{\lambda} \leq \lambda_i \leq \bar{\lambda} \\ 0 & \text{if } \tau_2 < F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j < \tau_1 \\ k_\lambda (1 - \lambda_i) & \text{if } F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j \leq \tau_2 \end{cases} \quad (20)$$

where  $\alpha > 0$  representing the scaling velocity,  $k_\lambda > 0$  and  $\tau_1 > \tau_2 > 0$  are two constants set manually to avoid over-scaling caused possibly by noise or disturbance. To avoid internal collisions among agents, a lower limit  $0 < \underline{\lambda} < 1$  is given for the scaling factor, such that the adjustment of  $\lambda_i$  should satisfy that  $\lambda_i \geq \underline{\lambda}$ . Similarly, to avoid the break of the communication network in MAS, an upper limit  $1 < \bar{\lambda} < \infty$  is set subject to the communication range of an agent such that the adjustment of  $\lambda_i$  should satisfy that  $\lambda_i \leq \bar{\lambda}$ . We will elaborate this criteria in detail in the subsequent section.

**Assumption 6.** There exists a constant  $\zeta_2$  such that  $0 \leq |\dot{\lambda}_i| \leq \zeta_2 < \infty$ .

**Remark 2.** The communication limitations investigated in this study include the limited communication range as well as the limited communication bandwidth. First, the limited communication range is reflected by introducing the adjacency matrix  $A = [a_{ij}]$  and the adjacency vector of the leader  $B = [b_i]$  into the control laws. The coordinated formation is achieved based on the information exchanged among agents within

the communication range. Second, the limited communication bandwidth is considered in the switching scheme of diverse control laws to promote the efficiency in communication bandwidth. Comparing Eq.(8) with Eq.(14), it can be inferred that the communication bandwidth required in the leader tracking mode is much less than the one in the coordination mode. In the leader tracking mode, the leader only needs to subscribe the triggering indicator occupying one bite, while in the coordination mode, the leader will subscribe the state of neighbor agents. Moreover, the state of spatial constraints does not need to be broadcasted. Collisions can be coordinately avoided by only broadcasting agents' state.

**Remark 3.** The role-switching-based coordination for collision avoidance is an explicit strategy of the event-triggered control, where the detection of the unexpected spatial constraints is defined as an event to trigger the role switching. If there are not communication limitations on range or bandwidth, the switching of role will not be needed in the coordination mechanism, since all agents could share the unlimited mutual information. However, communication among the agents often occurs over a wireless medium with finite capacity, so the role needs to be switched under event-triggered protocols.

#### IV. CONTROL PERFORMANCE ANALYSIS

In this section, we will prove the effectiveness of the proposed strategies in two cases: free space and constrained space.

**Case 1:** Trajectory tracking and formation following in a free space

**Theorem 1.** Consider  $N$  mobile agents with similar dynamics Eq.(1), moving in a free space, with Assumptions 1-5. There is one leader knowing the information of the given trajectory with the control law Eq.(8) and the rest of followers knowing the information of displacement to achieve the desired geometric formation under the control law Eq.(9) with bounded formation errors. The asymptotic stability of MAS can be acquired.

*Proof of Theorem 1:* See Appendix. ■

**Case 2:** Trajectory tracking and formation following in a constrained space

**Theorem 2.** Consider  $N$  mobile agents with similar dynamics Eq.(1), moving in a constrained space, with Assumptions 1-6. There is one leader knowing the information of the given trajectory with the control law Eq.(16) and the rest of followers knowing the information of displacement to achieve the desired geometric formation under the control law Eq.(17) with bounded formation errors. The asymptotic stability of MAS can be acquired if the following conditions are satisfied:

$$1) \quad F_1^c \dot{y}^d \geq 0 \quad (21)$$

$$2) \quad \hat{\Xi}^T (B \mathbf{1} \otimes \mathbf{I}_m) (k_c F_1^c - k_t \varepsilon_1) \geq 0 \quad (22)$$

where  $\Xi = [\varepsilon_2^T, \varepsilon_3^T, \dots, \varepsilon_N^T]^T$ , and  $\hat{\Xi} = [\hat{\varepsilon}_2^T, \hat{\varepsilon}_3^T, \dots, \hat{\varepsilon}_N^T]^T$ .

*Proof of Theorem 2:* See Appendix. ■

**Remark 4.** If the condition in Eq.(21) cannot be satisfied for a continuous duration  $t_{\max}$ , it means that the desired trajectory is unreasonable, although the basic assumption in this paper is that the designed trajectory does not have too much serious conflict with spatial constraints. In this case, the tracking task may be given up. On the other hand, if the condition that  $\underline{\lambda} \leq \lambda_i \leq \bar{\lambda}$  cannot be satisfied in the process of formation scaling, it means that the current formation shape cannot pass the restricted region. A possible solution is to switch into another formation shape and enter the emergency avoidance mode.

**Remark 5.** We do not pursue the consensus of  $\lambda_i, i = 2, \dots, N$  for two reasons. First, local formation scaling will benefit rapid collision avoidance and formation recovery. Second, considering various environmental conditions, the size of formation for some agents in MAS may need to be scaled up while some need to be scaled down to pass a restricted region without collisions. In this situation, a consensus of scaling factor would lead to failure of the task.

**Remark 6.** In the proposed control framework of the formation tracking, all agents including the leader and followers are a cooperative system. In the normal tracking mode, the task of trajectory tracking and the formation following are separate tasks, which will not be mutually influenced. However, the leader are always collaborating with followers to subscribe the triggering indicator. After switching to the coordination mode, the state feedback link from followers to the leader will be established, where the task of the trajectory tracking and the formation following will be mutually influenced to achieve the obstacles avoidance during formation tracking.

#### V. SIMULATION STUDIES

In this section, several simulation examples are presented to illustrate the effectiveness of the methods proposed in this paper. We consider a group of agents with  $N = 5$  to achieve a desired formation as a regular pentagon. The communication range is set as a circle with radius of  $3m$ .

##### A. Formation tracking in a 2-D free space

In the first simulation, the initial positions of  $N$  agents are initialized randomly within a circle range with radius  $r = 2m$ . we study the effectiveness of control laws Eqs.(8-9) for stationary formation control. Simulation results are shown in Figs.2-3.

Fig. 2 shows that agents are able to achieve the given formation from random initial positions. Fig. 3 shows that the formation errors converge to zeros along with the time indicating the success of formation generation.

##### B. Formation tracking in simple constrained space

In the second simulation, the performance of control laws Eqs.(16-17) are tested in a simple constrained space. Two sphere obstacles are considered with centers at  $x = [4 \quad -1 \quad 1.7]^T$  and  $x = [4 \quad -1 \quad -1.7]^T$ , respectively, and

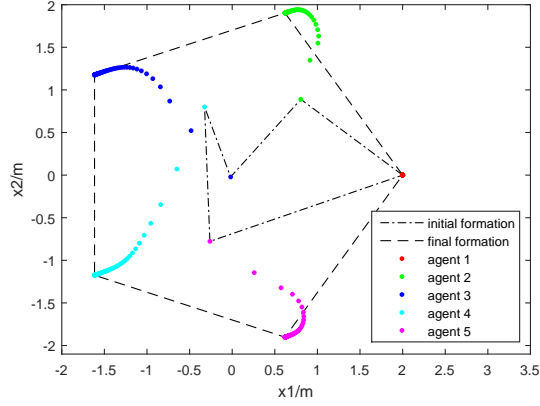


Fig. 2: Formation generation in the first simulation

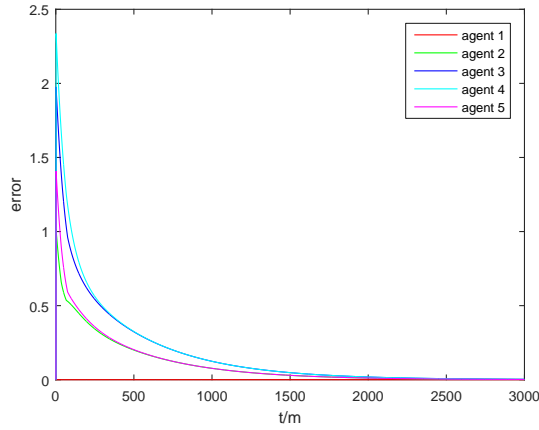


Fig. 3: Formation tracking error in the first simulation

the radius of  $2.5m$ . The desired trajectory for the leader is set as  $y^d(t) = [1.1t \ 2 \ 0]^T$ . Meanwhile, concerning the noise existed in the communication channel and sensor uncertainties, a set of white Gaussian noise series with the variance  $\sigma = 0.1$  is added to the neighbors' positions and the estimated spatial constraints. Simulation results are shown in Figs. 4-7.

Figs.4-5 show that all agents can successfully track the desired trajectory by locally adjusting the predefined trajectory to avoid collisions with obstacles. The tracking error for the agent  $v_1$  and formation errors for agents  $v_2-v_5$  (in Fig. 6) increase when agents get close to obstacles, and decrease to zeros when being far away from obstacles. Scaling factors are plotted in Fig.7.  $\lambda_3$  and  $\lambda_4$  indicate that the desired formation is scaled down to rapidly pass the restricted region without collisions. All results also indicate that the sensor uncertainties and communication noise have little influence on the formation tracking and obstacle avoidance, which shows the robustness of our method.

### C. Formation tracking in complex constrained spaces

In the third simulation, the effectiveness of control laws Eqs.(16) and (17) is further studied in a complex constrained space. As discussed in Section 3.3, two cases with two kinds

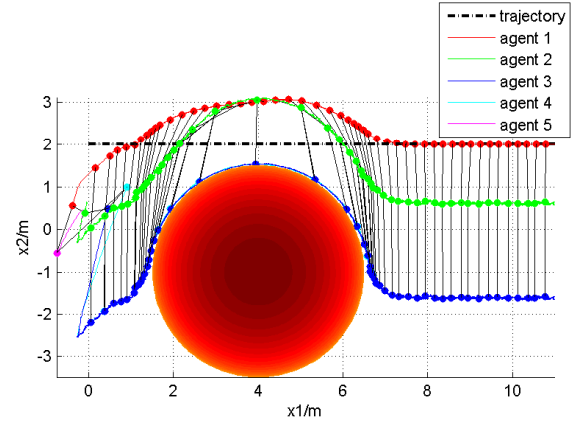


Fig. 4: Formation tracking, top view

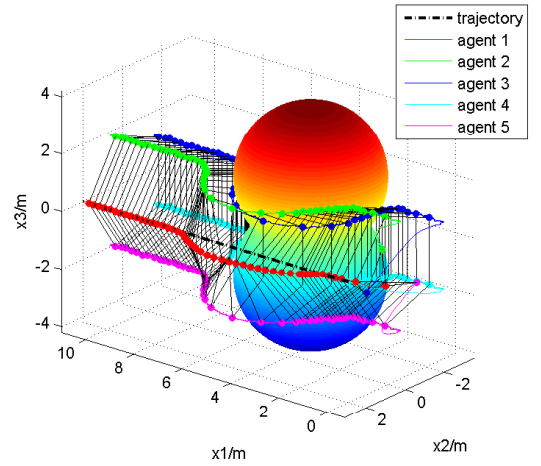


Fig. 5: Formation tracking, side view

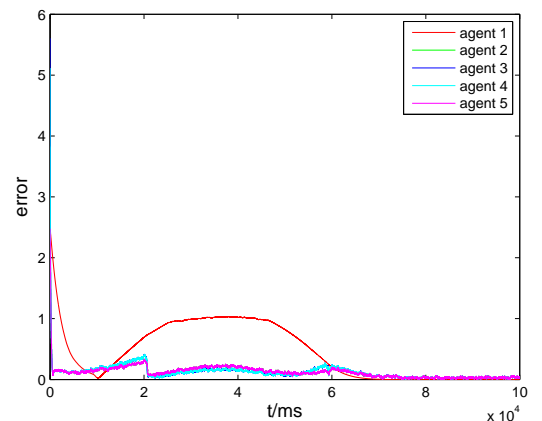


Fig. 6: Formation tracking error in the second simulation

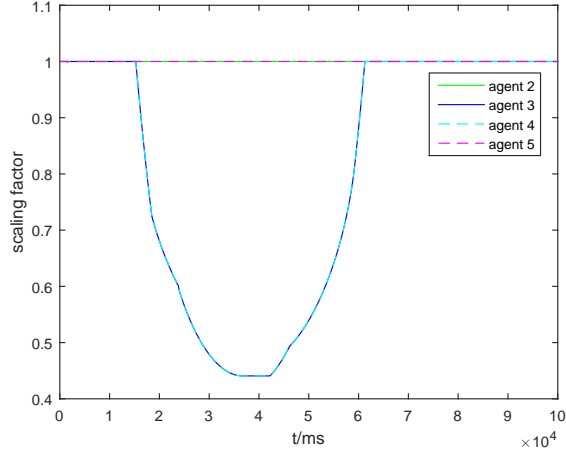


Fig. 7: Scaling factor in the second simulation

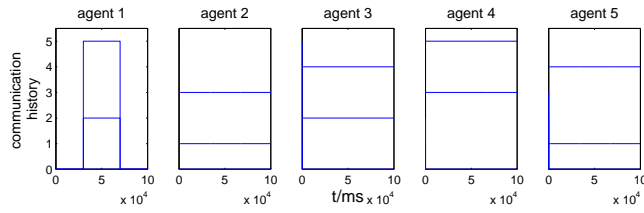


Fig. 8: Communication topology history

of spacial constraints are considered. The desired trajectory for the leader is set as  $y^d(t) = [t \ 2 \ 0]^T$ . The communication flow history of five agents are shown in Fig.8 to reflect the communication relationship among agents. Simulation results are shown in Figs.9-16.

Figs.9-10 & Figs.13-14 indicate that all agents can successfully track the desired trajectory while maintaining the predefined formation. Fig.11 & Fig.15 show that the scaling formation errors for agents  $v_i, i = 2, \dots, 5$  converge to zeros gradually. The tracking error for the agent  $v_1$  decreases to zero in the first phase but increases to a constant when entering the influence range of the obstacle and decreases to zero once again after leaving the influence range of the obstacle. The size of formation shape is scaled down in the first case and scaled up in the second case to enable that all agents keep a safe distance from the obstacles when entering the influence range of spatial constraints. Fig.12 & Fig.16 describe the variation tendency of formation scaling factors in two cases.

## VI. CONCLUSION

In this paper, the formation tracking control of the MAS with communication limitations in a constrained space has been studied based on predefined displacement. A role of coordinator has been introduced and a coordination approach has been developed to guarantee obstacle avoidance as well as formation keeping. Controllers have been designed according to the locally-exchanged information and locally-detected environmental constraints. A scaling factor has been proposed to scale up or scale down the given formation size in the case that the restricted path is impassable for the desired formation

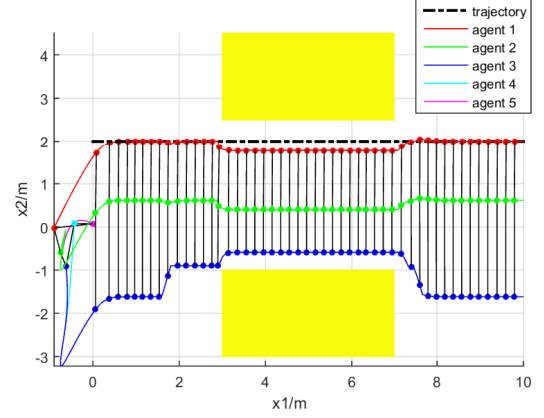


Fig. 9: Formation tracking with size adaptation in the first case, top view

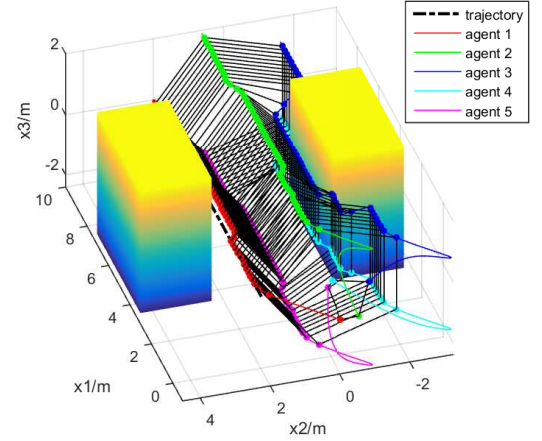


Fig. 10: Formation tracking with size adaptation in the first case, side view

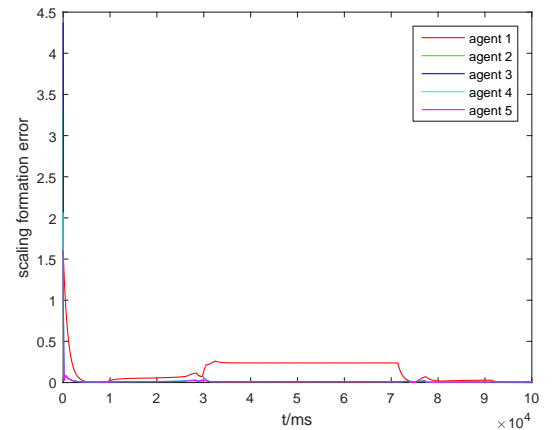


Fig. 11: Formation tracking error in the first case, in the third simulation



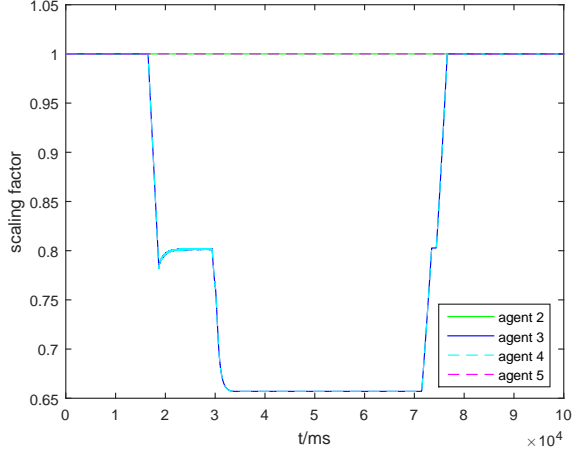


Fig. 12: Scaling factor in the first case, in the third simulation

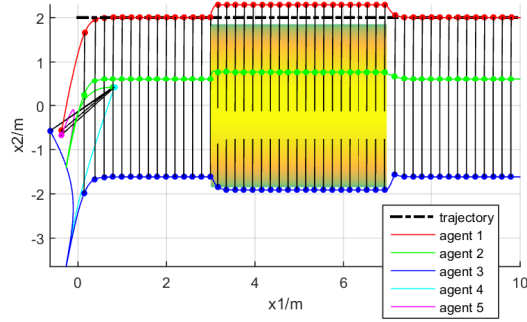


Fig. 13: Formation tracking with size adaptation in the first case, top view

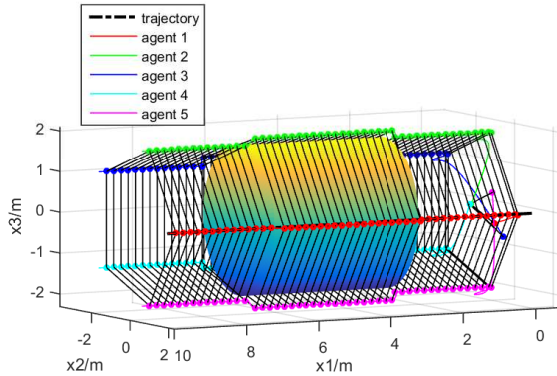


Fig. 14: Formation tracking with size adaptation in the first case, side view

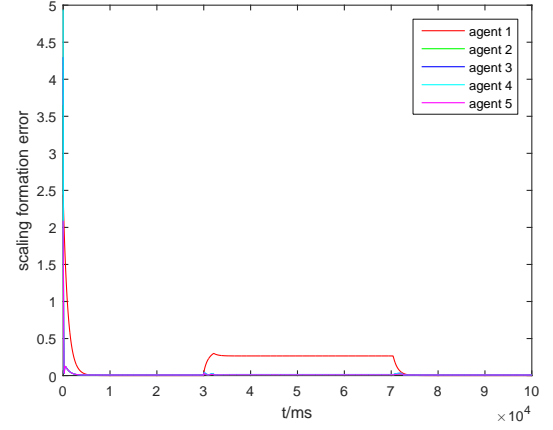


Fig. 15: Formation tracking error in the second case, in the third simulation

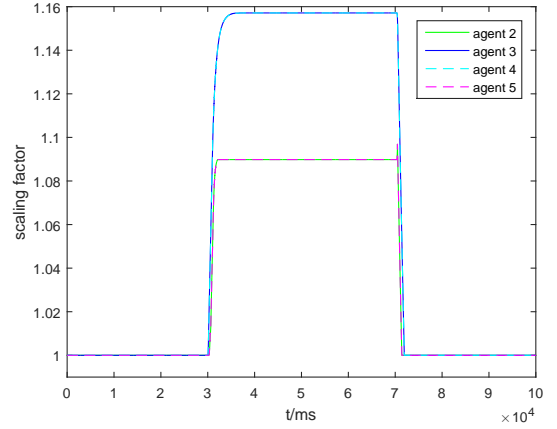


Fig. 16: Scaling factor in the second case, in the third simulation

and the adaptation law has been developed. The conditions for asymptotic stability of MAS have been discussed based on the Lyapunov theory. Finally, simulation results have been presented to illustrate the performance of proposed approaches. For the proposed schemes to be successfully applied in practical applications, many issues must be further considered, such as unknown/uncertain system dynamics [32] or physical constraints [33], which will be investigated in our future works.

Compared with the existing studies on formation control in constrained environments (e.g., [23], [28]), the role-switching strategy has been proposed under the condition of communication constraints for the sake of efficiency of event-triggered control in communication bandwidth, energy consumption and processor usage. Meanwhile, a feedback mechanism from followers to the leader has been built to promote the mutual coordination in MAS such that the failure case of following in [23] can be avoided. Compared with the achievements on the formation scaling reported in [27], the results have been further studied concerning the circumstance in a non-omniscient bounded space. The existence of unexpected spatial constraints

has been taken into consideration by adapting the scaling factor online in order to pass the region without collisions. In contrast with [34], which also proposed a switching strategy including a scheme for regular leader-follower formation and a scheme for obstacle avoidance, this paper considers the communication constraints and constructs the information flow that can ensure the success of formation tracking and promote the communication efficiency.

## APPENDIX

### *Proof of Theorem 1:*

#### **Step 1: Leader tracking**

The time derivative of the tracking error for  $v_1$  is

$$\dot{\varepsilon}_1 = \dot{x}_1 - \dot{y}^d = u_1 - \dot{y}^d \quad (23)$$

Substituting Eq.(8) into Eq.(23), we obtain  $\dot{\varepsilon}_1 = -k_t \varepsilon_1$ . The Lyapunov candidate for leader tracking can be chosen as  $V_1 = \frac{1}{2} \varepsilon_1^T \varepsilon_1$ . The derivative of  $V_1$  is

$$\dot{V}_1 = \dot{\varepsilon}_1^T \varepsilon_1 \quad (24)$$

Substituting Eq.(23) into Eq.(24), we have  $\dot{V}_1 = -k_t \varepsilon_1^T \varepsilon_1 \leq 0$ . Therefore,  $0 \leq V_1(t) \leq V_1(0)$ . We can conclude that the leader is able to track the desired trajectory in a free space.

#### **Step 2: Follower in formation**

Without considering flexible formation scaling in the free space, the control law for followers  $v_i, i = 2, \dots, N$ , in Eq.(17) can be rewritten as

$$\begin{aligned} u_i = & -k_n \frac{b_i}{d_i} \varepsilon_i - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} (\varepsilon_i - \varepsilon_j) \\ & -k_g g \left( \frac{b_i}{d_i} \varepsilon_i + \sum_{j=2}^N \frac{a_{ij}}{d_i} (\varepsilon_i - \varepsilon_j) \right) \end{aligned} \quad (25)$$

The time derivative of the formation following error for agent  $v_i, i = 2, \dots, N$ , can be written as

$$\dot{\varepsilon}_i = \dot{x}_i - \dot{x}_1 = u_i - u_1 \quad (26)$$

Substituting Eq.(25) into Eq.(26), we obtain

$$\begin{aligned} \dot{\varepsilon}_i = & -k_n \frac{b_i}{d_i} \varepsilon_i - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} (\varepsilon_i - \varepsilon_j) + k_t \varepsilon_1 \\ & -k_g g \left( \frac{b_i}{d_i} \varepsilon_i + \sum_{j=2}^N \frac{a_{ij}}{d_i} (\varepsilon_i - \varepsilon_j) \right) - \dot{y}^d \end{aligned} \quad (27)$$

Writing  $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbb{R}^{N-1}$ , the derivative of  $\Xi$  can be obtained as

$$\begin{aligned} \dot{\Xi} = & -k_n [\Upsilon(L+B) \otimes \mathbf{I}_m] \Xi - k_g \mathbf{I}_{m(N-1)} G(\Xi) \\ & + k_t (\mathbf{1} \otimes \mathbf{I}_m) \varepsilon_1 - (\mathbf{1} \otimes \mathbf{I}_m) \dot{y}^d \end{aligned} \quad (28)$$

where

$$\begin{aligned} G(\Xi) & \triangleq \begin{bmatrix} g \left( \frac{1}{d_2} b_2 \varepsilon_2 + \frac{1}{d_2} \sum_{j=2}^N \mathcal{L}_{2j} \varepsilon_j \right) \\ \vdots \\ g \left( \frac{1}{d_N} b_N \varepsilon_N + \frac{1}{d_N} \sum_{j=2}^N \mathcal{L}_{Nj} \varepsilon_j \right) \end{bmatrix} \\ & = \begin{bmatrix} g \left( \sum_{j=2}^N H_{j2} \varepsilon_j \right) \\ \vdots \\ g \left( \sum_{j=2}^N H_{jN} \varepsilon_j \right) \end{bmatrix} \end{aligned} \quad (29)$$

Choose the Lyapunov candidate for followers as

$$V_2 = \frac{1}{2} \Xi^T [\Upsilon(L+B) \otimes \mathbf{I}_m] \Xi \quad (30)$$

Then the derivative of  $V_2$  can be represented as

$$\begin{aligned} \dot{V}_2 & = \Xi^T (H \otimes \mathbf{I}_m) \dot{\Xi} \\ & = -k_n \Xi^T (H^2 \otimes \mathbf{I}_m) \Xi - k_g \Xi^T [H \otimes \mathbf{I}_m] G(\Xi) \\ & \quad + k_t \Xi^T (H \mathbf{1} \otimes \mathbf{I}_m) \varepsilon_1 - \Xi^T [H \mathbf{1} \otimes \mathbf{I}_m] \dot{y}^d \\ & \leq - \left( k_n - \frac{k_t}{2} \right) \Xi^T (H^2 \otimes \mathbf{I}_m) \Xi \\ & \quad - k_g \Xi^T (H \otimes \mathbf{I}_m) G(\Xi) + \frac{k_t(N-1)}{2} \varepsilon_1^T \varepsilon_1 \\ & \quad + \zeta_1 \sum_{i=2}^N \sum_{j=2}^N \|H_{ij} \varepsilon_i\| \end{aligned} \quad (31)$$

where we have

$$\begin{aligned} & \Xi^T (H \otimes \mathbf{I}_m) G(\Xi) \\ & = \left[ \sum_{j=2}^N H_{j2} \varepsilon_j^T \quad \dots \quad \sum_{j=2}^N H_{jN} \varepsilon_j^T \right] \times \\ & \quad \begin{bmatrix} \frac{\sum_{j=2}^N H_{j2} \varepsilon_j}{\|\sum_{j=2}^N H_{j2} \varepsilon_j\|} \\ \vdots \\ \frac{\sum_{j=2}^N H_{jN} \varepsilon_j}{\|\sum_{j=2}^N H_{jN} \varepsilon_j\|} \end{bmatrix} = \sum_{i=2}^N \sum_{j=2}^N \|H_{ij} \varepsilon_i\| \end{aligned} \quad (32)$$

It indicates that

$$\begin{aligned} \dot{V}_2 & \leq -k_n \Xi^T (H \otimes \mathbf{I}_m)^2 \Xi + \frac{k_t(N-1)}{2} \varepsilon_1^T \varepsilon_1 \\ & \quad - (k_g - \zeta_1) \sum_{i=2}^N \sum_{j=2}^N \|H_{ij} \varepsilon_i\| \end{aligned} \quad (33)$$

Under the condition that  $k_g \geq \zeta_1$ , there is

$$\begin{aligned} \dot{V}_2 & \leq -k_n \Xi^T (H \otimes \mathbf{I}_m)^2 \Xi + \frac{k_t(N-1)}{2} \varepsilon_1^T \varepsilon_1 \\ & = -k_n \Xi^T (H \otimes \mathbf{I}_m)^2 \Xi + k_t(N-1) V_1 \end{aligned} \quad (34)$$

From the *Step 1* in the *Proof of Theorem 1*, we can obtain that

$$\begin{aligned} \dot{V}_2 & \leq -k_n \Xi^T (H \otimes \mathbf{I}_m)^2 \Xi + k_t(N-1) V_1(0) \\ & = -c_1 V_2 + \mu \end{aligned} \quad (36)$$

where  $\mu = k_t(N-1) V_1(0)$ . Let  $\varrho \triangleq \mu/c_1$ , it follows that  $0 \leq V_2(t) \leq [V_2(0) - \varrho] e^{-c_1 t} + \varrho \leq V_2(0) + \varrho$ .  $V_1(t)$  and  $V_2(t)$  are proved to be both bounded. It shows that the tracking and formation errors will eventually converge to a constant.

**Proof of Theorem 2: Step 1: Leader tracking**

The control for the leader in Eq.(16) can be rewritten as

$$u_1 = \dot{y}^d - k_t \varepsilon_1 + k_c F_1^c + k_n \sum_{j=2}^N \frac{b_j}{d_1} \hat{\varepsilon}_j \operatorname{sgn} \left( \sum_{j=2}^N b_j \hat{\varepsilon}_j^T (k_c F_1^c - k_t \varepsilon_1) \right) \quad (37)$$

Choose the Lyapunov candidate for the leader as below

$$V_3 = \frac{1}{2} k_t \varepsilon_1^T \varepsilon_1 + U_1^c \quad (38)$$

The time derivative of  $V_3$  can be written as

$$\begin{aligned} \dot{V}_3 &= k_t \varepsilon_1^T \dot{\varepsilon}_1 - k_c F_1^{cT} \dot{x}_1 \\ &= (k_t \varepsilon_1 - k_c F_1^c)^T u_1 - k_t \varepsilon_1^T \dot{y}^d \end{aligned} \quad (39)$$

Substitute Eq.(37) into Eq.(39), then we have

$$\begin{aligned} \dot{V}_3 &= -\|k_t \varepsilon_1 - k_c F_1^c\|^2 - F_1^{cT} \dot{y}^d \\ &\quad - k_n \left\| \hat{\varepsilon}^T \left( \frac{1}{d_1} B \mathbf{1} \otimes I_m \right) (k_c F_1^c - k_t \varepsilon_1) \right\| \end{aligned} \quad (40)$$

If the condition in Eq.(21) is satisfied, we have  $\dot{V}_3 < 0$ ,  $0 \leq V_3(t) \leq V_3(0)$ , then the asymptotic stability can be easily obtained [35]. The boundedness of  $\varepsilon_1$  and  $F_1^c$  can be guaranteed and we may assume that  $0 \leq \|\varepsilon_1\| \leq \phi_e < \infty$  and  $0 \leq \|F_1^c\| \leq \phi_f < \infty$ .

Otherwise, if the condition in Eq.(21) is not satisfied, we have  $F_1^{cT} \dot{y}^d < 0$ , indicating that the leader agent is getting close to an obstacle. In this case, we may set a tolerable upper limit  $t_{\max}$  for the continuous duration such that  $F_1^{cT} \dot{y}^d < 0$ . This can be a criterion in realistic tasks to indicate whether the tracking task should be given up or the desired trajectory should be replanned. The basic assumption of this paper is that the designed trajectory for the leader does not have too much serious conflict with spatial constraints. However, if the duration to make that the condition  $F_1^{cT} \dot{y}^d \geq 0$  cannot be satisfied exceeds  $t_{\max}$ , which may be caused by various uncertainties or an unreasonable given trajectory, the formation tracking task may be given up or the desired trajectory should be redesigned.

**Step 2: Follower in formation**

The controllers for followers in Eq.(17) can be rewritten as

$$\begin{aligned} u_i &= -k_n \frac{b_i}{d_i} \hat{\varepsilon}_i - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} \hat{\varepsilon}_{ij} + k_c F_i^c \\ &\quad - k_g g \left( \frac{b_i}{d_i} \hat{\varepsilon}_i + \sum_{j=2}^N \frac{a_{ij}}{d_i} \hat{\varepsilon}_{ij} \right) \end{aligned} \quad (41)$$

Thus, the derivative of the scaling formation following errors for followers  $v_i, i = 2, \dots, N$  can be represented as

$$\begin{aligned} \dot{\hat{\varepsilon}}_i &= \dot{x}_i - \dot{x}_1 - \dot{\lambda}_i \delta_i \\ &= -k_n \frac{b_i}{d_i} \hat{\varepsilon}_i - k_n \sum_{j=2}^N \frac{a_{ij}}{d_i} \hat{\varepsilon}_{ij} + k_c F_i^c - u_1 \\ &\quad - k_g g \left( \frac{b_i}{d_i} \hat{\varepsilon}_i + \sum_{j=2}^N \frac{a_{ij}}{d_i} \hat{\varepsilon}_{ij} \right) - \dot{\lambda}_i \delta_i \end{aligned} \quad (42)$$

where  $\hat{\varepsilon}_{ij} = \hat{\varepsilon}_i - \hat{\varepsilon}_j = x_i - x_j - (\lambda_i \delta_i - \lambda_j \delta_j)$ . Denote  $B_2$  with  $B_{2ij} = b_{ij}$ ,  $\Lambda = \operatorname{diag}(\lambda_2, \lambda_3, \dots, \lambda_N)$  and  $F^c = [F_2^{cT}, F_3^{cT}, \dots, F_N^{cT}]^T$ , we obtain

$$\begin{aligned} \dot{\hat{\varepsilon}} &= -k_n (\Upsilon B \otimes I_m) \hat{\varepsilon} - k_n (\Upsilon L \otimes I_m) \hat{\varepsilon} \\ &\quad + k_c I_{m(N-1)} F^c - k_g I_{m(N-1)} G(\hat{\varepsilon}) \\ &\quad - (\mathbf{1} \otimes I_m) u_1 - (\dot{\Lambda} \otimes I_m) \Delta \\ &= -k_n (H \otimes I_m) \hat{\varepsilon} + k_c I_{m(N-1)} F^c - (\mathbf{1} \otimes I_m) u_1 \\ &\quad - k_g I_{m(N-1)} G(\hat{\varepsilon}) - (\dot{\Lambda} \otimes I_m) \Delta \end{aligned} \quad (43)$$

where  $\Delta = [\delta_2^T, \delta_3^T, \dots, \delta_N^T]^T$ . Substituting Eq.(37) into Eq.(43) under the condition in Eq.(22), we have

$$\begin{aligned} \dot{\hat{\varepsilon}} &= -k_n (H \otimes I_m) \hat{\varepsilon} - (\mathbf{1} \otimes I_m) (\dot{y}^d - k_t \varepsilon_1 + k_c F_1^c) \\ &\quad + k_c I_{m(N-1)} F^c - k_g I_{m(N-1)} G(\hat{\varepsilon}) \\ &\quad - \frac{k_n}{d_1} (B_2 \otimes I_m) \hat{\varepsilon} - (\dot{\Lambda} \otimes I_m) \Delta \end{aligned} \quad (44)$$

Choose the Lyapunov candidate for followers as below

$$V_4 = \frac{1}{2} \hat{\varepsilon}^T (H \otimes I_m) \hat{\varepsilon} + \frac{1}{k_c} \sum_{i=2}^N U_i^c \quad (45)$$

The time derivative of  $V_4$  can be written as

$$\dot{V}_4 = \hat{\varepsilon}^T (H \otimes I_m) \dot{\hat{\varepsilon}} - \sum_{i=2}^N F_i^{cT} \dot{x}_i \quad (46)$$

with

$$\begin{aligned} &\hat{\varepsilon}^T (H \otimes I_m) \dot{\hat{\varepsilon}} \\ &= -k_n \hat{\varepsilon}^T (H^2 \otimes I_m) \hat{\varepsilon} + k_c \hat{\varepsilon}^T (H \otimes I_m) F^c \\ &\quad - k_g \hat{\varepsilon}^T (H \otimes I_m) G(\hat{\varepsilon}) - \hat{\varepsilon}^T (H \dot{\Lambda} \otimes I_m) \Delta \\ &\quad - \hat{\varepsilon}^T (H \mathbf{1} \otimes I_m) (\dot{y}^d - k_t \varepsilon_1 + k_c F_1^c) \\ &\quad - \frac{k_n}{d_1} \hat{\varepsilon}^T (H B_2 \otimes I_m) \hat{\varepsilon} \end{aligned} \quad (47)$$

$$- \sum_{i=2}^N F_i^{cT} \dot{x}_i = -k_c \sum_{i=2}^N \|F_i^c\|^2 + k_g F^{cT} G(\hat{\varepsilon}) + k_n F^{cT} (H \otimes I_m) \hat{\varepsilon} \quad (48)$$

Thus, we obtain

$$\begin{aligned} \dot{V}_4 &= -k_n \hat{\varepsilon}^T (H^2 \otimes I_m) \hat{\varepsilon} - k_c \sum_{i=2}^N \|F_i^c\|^2 \\ &\quad - \frac{k_n}{d_1} \hat{\varepsilon}^T (H B_2 \otimes I_m) \hat{\varepsilon} + k_g F^{cT} G(\hat{\varepsilon}) \\ &\quad - \hat{\varepsilon}^T (H \mathbf{1} \otimes I_m) (\dot{y}^d + k_c F_1^c - k_t \varepsilon_1) \\ &\quad + (k_c + k_n) \hat{\varepsilon}^T (H \otimes I_m) F^c \\ &\quad - k_g \hat{\varepsilon}^T (H \otimes I_m) G(\hat{\varepsilon}) - \hat{\varepsilon}^T (H \dot{\Lambda} \otimes I_m) \Delta \end{aligned} \quad (49)$$

Given a desired formation,  $\|\Delta\|$  is a constant. Define  $\zeta_3 =$

$\zeta_1 + \zeta_2 \|\Delta\| + \phi_e + \phi_f$ , then we have

$$\begin{aligned} \dot{V}_4 \leq & -k_n \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} - k_c \sum_{i=2}^N \|F_i^c\|^2 \\ & - \frac{k_n}{d_1} \hat{\Xi}^T (HB_2 \otimes I_m) \hat{\Xi} + \zeta_3 \sum_{i=2}^N \sum_{j=2}^N \|H_{ij} \hat{\varepsilon}_i\| \\ & + (k_c + k_n) \hat{\Xi}^T (H \otimes I_m) F^c \\ & - k_g \hat{\Xi}^T (H \otimes I_m) G(\hat{\Xi}) + k_g F^{cT} G(\hat{\Xi}) \end{aligned} \quad (50)$$

Following the same procedure in Eq.(32), we obtain

$$\begin{aligned} & -\hat{\Xi}^T (H \mathbf{1} \otimes I_m) (\dot{y}^d + k_c F_1^c - k_t \varepsilon_1) \\ & -\hat{\Xi}^T (H \dot{\Lambda} \otimes I_m) \Delta - k_g \Xi^T (H \otimes I_m) G(\Xi) \\ = & - (k_g - \zeta_3) \sum_{i=2}^N \sum_{j=2}^N \|H_{ij} \varepsilon_i\| \end{aligned} \quad (51)$$

Guaranteeing  $k_g \geq \zeta_3$ , we have

$$\begin{aligned} & -\hat{\Xi}^T (H \dot{\Lambda} \otimes I_m) \Delta - k_g \Xi^T (H \otimes I_m) G(\Xi) \\ & -\hat{\Xi}^T (H \mathbf{1} \otimes I_m) (\dot{y}^d + k_c F_1^c - k_t \varepsilon_1) \leq 0 \end{aligned} \quad (52)$$

Thus, we obtain

$$\begin{aligned} \dot{V}_4 \leq & -k_n \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} - k_c \sum_{i=2}^N \|F_i^c\|^2 + k_g F^{cT} G(\hat{\Xi}) \\ & - \frac{k_n}{d_1} \hat{\Xi}^T (HB_2 \otimes I_m) \hat{\Xi} + (k_c + k_n) \hat{\Xi}^T (H \otimes I_m) F^c \end{aligned} \quad (53)$$

Considering  $\|G(\hat{\Xi})\| = \sqrt{N-1}$ , we have

$$\begin{aligned} \dot{V}_4 \leq & -k_n \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} - k_c \sum_{i=2}^N \|F_i^c\|^2 \\ & - \frac{k_n}{d_1} \hat{\Xi}^T (HB_2 \otimes I_m) \hat{\Xi} + k_g \sqrt{N-1} \sum_{i=2}^N \|F_i^c\| \\ & + (k_c + k_n) \hat{\Xi}^T (H \otimes I_m) F^c \end{aligned} \quad (54)$$

With the conditions  $\|\varepsilon_1\| \leq \phi_e$  and  $\|F_1^c\| \leq \phi_{f1}$  which have been proved above, we obtain

$$\begin{aligned} & \hat{\Xi}^T (H \otimes I_m) F^c \\ \leq & \frac{1}{2} \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} + \frac{1}{2} \sum_{i=2}^N \|F_i^c\|^2 \end{aligned} \quad (55)$$

$$k_g \sqrt{N-1} \sum_{i=2}^N \|F_i^c\| - \varrho \sum_{i=2}^N \|F_i^c\|^2 \leq \psi \quad (56)$$

where  $\psi > 0$  is a positive scalar, then we obtain

$$\begin{aligned} \dot{V}_4 \leq & -\frac{k_n}{2} \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} - \left(k_c - \frac{k_n}{2}\right) \sum_{i=2}^N \|F_i^c\|^2 \\ & - \frac{k_n}{d_1} \hat{\Xi}^T (HB_2 \otimes I_m) \hat{\Xi} + k_c \hat{\Xi}^T (H \otimes I_m) F^c + \psi \end{aligned} \quad (57)$$

When  $\hat{\Xi}^T (H \otimes I_m) F^c \leq 0$ , we set  $\dot{\lambda}_i = k_\lambda (1 - \lambda_i)$ ,  $i = 2, \dots, N$ . However, this condition is not available in distributed control strategy. Therefore, we have to strengthen this condi-

tion for every agent. Eq.(57) can be rewritten as

$$\begin{aligned} \dot{V}_4 \leq & -\frac{k_n}{2} \hat{\Xi}^T (H^2 \otimes I_m) \hat{\Xi} - \left(k_c - \frac{k_n}{2}\right) \sum_{i=2}^N \|F_i^c\|^2 \\ & - \frac{k_n}{d_1} \hat{\Xi}^T (HB_2 \otimes I_m) \hat{\Xi} + k_c \sum_{i=2}^N F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j + \psi \end{aligned} \quad (58)$$

With  $\hat{\varepsilon}_i = \varepsilon_i + (1 - \lambda_i) \delta_i$ , Eq.(58) can be rewritten as

$$\begin{aligned} & \sum_{i=2}^N F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j \\ = & \sum_{i=2}^N \left[ F_i^{cT} \sum_{j=2}^N H_{ij} \varepsilon_j + F_i^{cT} (1 - \lambda_i) H_{ii} \delta_i \right. \\ & \left. + F_i^{cT} \sum_{j=2, j \neq i}^N (1 - \lambda_j) H_{ij} \delta_j \right] \end{aligned} \quad (59)$$

For the agent  $v_i$  satisfying that  $F_i^{cT} \sum_{j=2}^N H_{ij} \hat{\varepsilon}_j \leq 0$ , we propose  $\dot{\lambda}_i = k_\lambda (1 - \lambda_i)$ ; otherwise, the adaptation of  $\lambda_i$  should guarantee that  $\lambda_i > 1$  when  $F_i^{cT} \delta_i > 0$  and  $\lambda_i < 1$  when  $F_i^{cT} \delta_i < 0$ . Therefore, we design the updating strategies for scaling factors  $\lambda_i, i = 2, \dots, N$  as Eq.(20). Then we have

$$\dot{V}_4 \leq -c_2 V_4 + \psi + k_c \hat{\Xi}^T (H \otimes I_m) F^c \quad (60)$$

To conclude, by introducing the size scaling adaptation matrix  $\Lambda$ ,  $k_c \Delta^T [(I_{N-1} - \Lambda) H \otimes I_m] F^c$  will limit the norm of  $k_c \hat{\Xi}^T (H \otimes I_m) F^c$  to a certain extent. ■

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